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MATHEMATICAL MODELS FOR STUDYING ENVIRONMENT POLLUTION RISKS

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The problem of long-term ecological prediction by means of mathematical modeling with available factual data on climate dynamics is discussed. The technique of quantitative estimates of risk/vulnerability on the basis of forward and inverse modeling and methods of the sensitivity theory is described. Examples of the calculated risk domains for Lake Baikal are given.

Key words: *environment pollution, environmental risk, atmosphere hydrodynamics, mathematical modeling.*

Introduction. The problem of long-term ecological prediction is at the interface of sciences of biospheric and climatic processes formed under the action of both natural and man-caused factors. Recent events have clearly demonstrated that ecology at the modern stage of social development in industrially loaded regions becomes an important social, economic, and geopolitical factors. Indeed, with increasing technical complexity and specific power of energetic and industrial objects, the hazard of large-scale man-caused catastrophes normally is not reduced. These catastrophes, in turn, can lead to severe ecological consequences. The entire set of environmental problems can be particularly urgent in Siberian regions where the national economy is mainly oriented to extraction of raw materials, and creation of powerful energy facilities for energy export is planned.

Modern trends of solving international and domestic conflicts with the use of hazardous means and armament systems and with deliberate destruction of ecologically hazardous civil objects also seem to be precarious from the viewpoint of ecology. In addition, there can arise some unexpected consequences of the action of both natural and man-caused loads from sources of heat, moisture, and pollutants. Since all these events occur on the background of natural processes of various time and space scales, it becomes necessary to formulate and solve a new class of problems: interrelated problems of ecology and climate.

Of fundamental importance in environmental research are the notions of risks and vulnerability of territories to natural and man-caused factors. They can be used to identify potentially hazardous situations and objects and quantitatively evaluate the degree of possible consequences of catastrophic events.

The special features of this class of problems is the necessity of considering a wide range of interacting processes over long time intervals in domains of different scales with uncertainties in external and internal sources of disturbances. The models of the processes and the data of observations necessary for solving the problems should be used jointly in the regime of direct and inverse relations. Today's models of the climatic system, models of transport and transformation of gaseous pollutants and aerosols, which form the theoretical basis for methods of ecological prediction, are rather complicated, since they should take into account both natural and man-caused factors affecting the processes considered [1–3]. The general issues of organization of climatic models and their application for studying changes in climate are described in [4, 5].

Since numerous processes of different scales and dissimilar factual information should be taken into account together, it is convenient to construct numerical models and algorithms for their implementation on the basis of variational principles in combination with methods of splitting and decomposition [6]. In this role, variational principles ensure agreement between various elements of the set of models in a manner that the models at all stages of computations retain the meaning implied in their initial formulations.

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The structure of mathematical models and methods designed for solving diagnostic and prognostic environmental problems is described in the present paper. Various aspects of man-caused factors are directly accounted in the models via the parametric description of the sources of heat, moisture, and pollutants and the changes in the Earth surface on large areas. The technique for constructing deterministic and deterministic-stochastic estimates of environmental risks for receptor regions is used; the technique is based on forward and inverse modeling, sensitivity theory of models, and mathematical theory of risk.

Models of Atmospheric Processes. The processes under study are described by models of hydrothermodynamics in the climatic system, models of transport and transformation of moisture and chemically and optically active gaseous pollutants and aerosols. The source functions in the models take into account the action of natural and man-caused factors. For comprehensive consideration of the model of the processes, the monitoring system, and the set of functionals for organizing interaction between them in the regime of direct and inverse relations, we assume that all elements of the system (i.e., models and observations) can contain uncertainties and errors. In this case, it is natural to consider construction of algorithms for such relations, based on the condition of minimization of a certain measure of uncertainties and errors.

We consider models from the set of models of the climate-ecological system [7, 8], which are directly related to the processes of transport of heat, moisture, and optically and chemically active substances in the atmosphere

$$L\boldsymbol{\varphi} \equiv \frac{\partial \pi \varphi_i}{\partial t} + \operatorname{div} \pi(\varphi_i \boldsymbol{u} - \mu_i \operatorname{grad} \varphi_i) + \pi(H(\boldsymbol{\varphi}))_i - \pi(f_i(\boldsymbol{x}, t) + r_i) = 0, \qquad i = \overline{1, n}. \tag{1}$$

Here $\boldsymbol{\varphi} = \{\varphi_i(\boldsymbol{x}, t),\ i = \overline{1, n}\} \in Q(D_t)$ is the vector function of state. Its components φ_i describe the potential temperature, ratios of the water–air mixture for humidity characteristics in the atmosphere (water vapor, cloud water, or rain water), and concentrations of gaseous pollutants and aerosols, D_t is the range of variation of spatial coordinates and time, $\boldsymbol{f} = \{f_i(\boldsymbol{x}, t),\ i = \overline{1, n}\}$ is the function of sources of heat, moisture, and pollutants, $\boldsymbol{r} = \{r_i(\boldsymbol{x}, t),\ i = \overline{1, n}\}$ are the functions that describe uncertainties and errors of the models, $\boldsymbol{u} = (u_1, u_2, u_3)$ is the velocity vector, $\mu_i = (\mu_1, \mu_2, \mu_3)_i$ are the coefficients of turbulent exchange for the substance φ_i in the direction of the coordinates $\boldsymbol{x} = \{x_i,\ i = \overline{1, 3}\}$, the form of the function π is determined by the structure of the vertical coordinate in the domain D_t, and $H(\boldsymbol{\varphi})$ is a nonlinear matrix operator that describes the local processes of transformation of the corresponding substances. This operator does not contain derivatives of the functions of state with respect to $\boldsymbol{x}$ and t. If the model takes into account the processes of aerosol formation and transformation, it contains one more variable: particle size. Then, the transformation operator has an integrodifferential structure in terms of this variable [9]. The functions $\boldsymbol{u}$, μ_i, and f_i and the input data of initial and boundary conditions are included into the set of components of the vector of parameters $\boldsymbol{Y}$, which belong to the range of admissible values $R(D_t)$. The structure of domains and coordinate systems described in detail in [8] are used here.

The initial conditions at $t = 0$ and model parameters can be written as

$$\boldsymbol{\varphi}^0 = \boldsymbol{\varphi}_a^0 + \xi(\boldsymbol{x}), \qquad \boldsymbol{Y} = \boldsymbol{Y}_a + \boldsymbol{\zeta}(\boldsymbol{x}, t), \tag{2}$$

where $\boldsymbol{\varphi}_a^0$ and $\boldsymbol{Y}_a$ are specified *a priori* estimates of the initial fields $\boldsymbol{\varphi}^0$ and the vector of parameters $\boldsymbol{Y}$; $\boldsymbol{\xi}(\boldsymbol{x})$ and $\boldsymbol{\zeta}(\boldsymbol{x}, t)$ are the errors and uncertainties in initial fields and parameters.

The boundary conditions for closing the model are determined by the physical content of the problem under study. Based on the form of the advective-diffusion operators in (1), it is convenient to present them in the form

$$\mu_n \frac{\partial \varphi_i}{\partial n} + \alpha_i \varphi_i - g_i = 0 \qquad (i = \overline{1, n}), \tag{3}$$

where $\alpha_i = \alpha_i(\boldsymbol{x}, t, \boldsymbol{\varphi})$ are the functions that determine the behavior of substance fluxes at the boundaries, including the regimes of their interaction with the Earth surface, and g_i are the source functions specified at the boundaries Ω_t of the domain D_t.

All numerical schemes and methods for solving the problems are constructed with the use of variational principles [6]. For this purpose, we write a variational formulation of the set of models (1)–(3) in the form of the integral identity

$$I(\boldsymbol{\varphi}, \boldsymbol{Y}, \boldsymbol{\varphi}^*) = \int (L(\boldsymbol{\varphi}), \boldsymbol{\varphi}^*)\, dD\, dt = 0, \tag{4}$$

where $\boldsymbol{\varphi}^*$ belongs to the space $Q^*(D_t)$ adjoint with respect to $Q(D_t)$. The integral identity (4) is transformed with allowance for the boundary and initial conditions so that it yields the relation of the energy balance for the system considered after the substitution $\boldsymbol{\varphi}^* = \boldsymbol{\varphi}$.

Performing, under this condition, all necessary transformations in (4) for model (1)–(3), we finally obtain the integral identity

$$I(\boldsymbol{\varphi}, \boldsymbol{Y}, \boldsymbol{\varphi}^*) \equiv \sum_{i=1}^{n} \Big\{ (\Lambda\varphi, \varphi^*)_i + \int\limits_{D_t} ((H(\boldsymbol{\varphi}))_i - f_i - r_i)\varphi_i^* \pi \, dD\, dt \Big\} = 0. \tag{5}$$

Here,

$$(\Lambda\varphi, \varphi^*)_i \equiv \Big\{ \int\limits_{D_t} \Big[\frac{1}{2} \Big(\Big(\varphi^* \frac{\partial \pi\varphi}{\partial t} - \varphi \frac{\partial \pi\varphi^*}{\partial t} \Big) + (\varphi^* \operatorname{div} \pi\varphi\boldsymbol{u} - \varphi \operatorname{div} \pi\varphi^*\boldsymbol{u}) \Big) + \pi\mu \operatorname{grad}\varphi \operatorname{grad}\varphi^* \Big] dD\, dt$$

$$+ \frac{1}{2} \int\limits_{D} \varphi\varphi^* \pi \, dD \Big|_0^{\bar{t}} + \int\limits_{\Omega_t} \Big(\frac{\varphi u_n}{2} + \alpha\varphi - g \Big) \varphi^* \pi \, d\Omega\, dt \Big\}_i \tag{6}$$

(u_n is the velocity-vector component normal to the boundary). The variational formulation (5), (6) is used to construct discrete approximations of the model.

The issues of practical implementation of the principal models of natural processes for the use in traditional regimes of forward modeling have been well developed [1–3, 6]. We use the variational organization of the models to construct, on their basis, combined methods of forward and inverse modeling for problems of a higher system level, associated with the issues of ecological safety and control of environment quality [7, 10, 11]. The principal elements of these methods are the algorithms of calculating functions of sensitivity to variations of input data, parameters, and sources. The algorithms, in turn, require some forward and adjoint problems to be solved. All these algorithmic structures are generated by the corresponding variational principles in which the key role belongs to the integral identity of the form (4).

Algorithms for Risk Estimates. Estimation of environmental risks and vulnerability of territories to man-caused factors is one of the typical problems of ecological prediction and design. For problems of this class, we use combined methods of forward and inverse modeling and methods of the sensitivity theory of models and generalized characteristics of environment quality. We briefly describe the main idea and the scheme of its implementation for practical purposes.

First, we determine the set of estimated characteristics in the form of the functionals

$$\Phi_k(\boldsymbol{\varphi}) = \int\limits_{D_t} F_k(\boldsymbol{\varphi}) \chi_k(\boldsymbol{x}, t) \, dD\, dt \qquad (k = \overline{1, K}, \quad K \geqslant 1), \tag{7}$$

where $F_k(\boldsymbol{\varphi})$ are functions of a given form, determined and differentiated on the set of values of the functions of state $Q(D_t)$, $\chi_k(\boldsymbol{x}, t) \geqslant 0$ are the weight functions that belong to the adjoint space $Q^*(D_t)$, and $\chi_k(\boldsymbol{x}, t)\, dD\, dt$ are the corresponding Radon and Dirac measures in D_t. Part of the domain $D_t^m \subset D_t$, where the weight function has nonzero values, is called a receptor, i.e., receiver of disturbances described by the function $F_k(\boldsymbol{\varphi})$. With a proper choice of the functions $F_k(\boldsymbol{\varphi})$ and χ_k, functionals of this form can describe the generalized characteristics of the behavior of the system, ecological restrictions on environment quality, results of observations of various types, control criteria, criteria of model quality, etc. [10].

For this set of functionals, we construct the main relations of the sensitivity theory, which determine the relations between the variations of $\delta\Phi_k(\boldsymbol{\varphi})$ and variations of the model parameters (see [12]):

$$\delta\Phi_k^h(\boldsymbol{\varphi}) \equiv \frac{\partial}{\partial\varepsilon} I^h(\boldsymbol{\varphi}, \boldsymbol{Y} + \varepsilon\delta\boldsymbol{Y}, \boldsymbol{\varphi}_k^*) \Big|_{\varepsilon=0} \equiv (\boldsymbol{\Gamma}_k, \delta\boldsymbol{Y}) \equiv \sum_{i=1}^{N} \Gamma_{ki}\, \delta Y_i$$

$$\equiv \sum_{i=1}^{n} \Big\{ (\delta\Lambda\varphi, \varphi_k^*)_i + \int\limits_{D_t} (\delta(H(\boldsymbol{\varphi}))_i - \delta f_i) \varphi_{ki}^* \pi \, dD\, dt \Big\}^h, \qquad k = \overline{1, K}; \tag{8}$$

$$(\delta\Lambda\varphi, \varphi_k^*)_i \equiv \Big\{ \int\limits_{D_t} \Big(\frac{1}{2} (\varphi_k^* \operatorname{div} \pi\varphi\, \delta\boldsymbol{u} - \varphi \operatorname{div} \pi\varphi_k^*\, \delta\boldsymbol{u}) + \pi\, \delta\mu \operatorname{grad}\varphi \operatorname{grad}\varphi_k^* \Big) dD\, dt$$

$$+ \frac{1}{2} \int\limits_{D} \delta\varphi\, \varphi_k^* \pi \, dD \Big|_{t=0} + \int\limits_{\Omega_t} \Big(\frac{\varphi\, \delta u_n}{2} + \delta\alpha\, \varphi - \delta g \Big) \varphi_k^* \pi \, d\Omega\, dt \Big\}_i^h. \tag{9}$$

Here ε is a real parameter; $\delta\boldsymbol{Y} = \{\delta Y_i\}$ are variations of model parameters and sources of external and internal factors; $\Gamma_k = \{\Gamma_{ki}\}$ are the functions of sensitivity of the functional $\Phi_k^h(\boldsymbol{\varphi})$ to these variations ($k = \overline{1,K}$, $i = \overline{1,N}$), $\boldsymbol{\varphi}$ is the solution of the basic problem and $\boldsymbol{\varphi}_k^*$ is the solution of the adjoint problem corresponding to the functional $\Phi_k^h(\boldsymbol{\varphi})$, which are generated by the variational principle from the stationary conditions of the discrete analog of the extended functional $\tilde{\Phi}_k^h(\boldsymbol{\varphi}) = \Phi_k(\boldsymbol{\varphi}) + I^h(\boldsymbol{\varphi}, \boldsymbol{\varphi}^*, \boldsymbol{Y})$ with respect to arbitrary independent variations of the functions $\boldsymbol{\varphi}$ and $\boldsymbol{\varphi}^*$ in the nodes of the grid domain D_t^h. The superscript h indicates the discrete analogs of the corresponding objects. Discretization operations are also applied to all functional arguments and to space–time domains. The algorithms for constructing the basic relation and sensitivity functions in problems of the considered class are described in [6, 7].

Terms containing sources of heat, moisture, and pollutants in formulas (8) and (9) should be specially noted. The factors at variations of the sources δf_i and δg_i are the corresponding sensitivity functions (SF). They are a measure of the direct influence of variations of the sources on the values of functional variations (in linear problems, the influence of the sources themselves on the value of the functional).

The sensitivity functions of functionals (7) with respect to variations of sources are determined in the domain D_t. Depending on the objectives of the study and for convenience of interpretation, they can be called the functions of influence or hazard of the sources, significance level, information value of the monitoring system, etc.

The supports of these functions can be interpreted as observability domains of the monitoring system located in the receptor domain. Using the terminology of the theory of differential equations, they can also be called the dependence domains and influence domains for the values of the functions of state of model (1)–(3) in the receptor.

The information meaning of the hazard function for the functionals determining the quality of atmosphere in the receptor can be described as follows. Its value at the point $(\boldsymbol{x}, t) \in D_t$ is the relative contribution of pollutant ejection by a source located at this point (in the time interval of its action) to the total value of pollutants entering the receptor atmosphere during the observation period, which is presented by the functional. Therefore, the sensitivity relations and sensitivity functions contain quantitative information for measurement of the degree of environmental risks for the receptor region and show the character of its vulnerability to disturbances from potentially hazardous sources.

The sensitivity functions are calculated via the solutions of the basic and adjoint problems for model (1)–(3) with undisturbed values of the input data and, therefore, have a deterministic character. Variations of model parameters, initial and boundary conditions, and sources can be either deterministic or random. For the sources, emergence of variations can be associated with the possibility of nonstandard situations. Such situations, if not deliberately stimulated, usually have a random character.

For quantitative estimation of environmental risks, we introduce some threshold values of variations of functionals (7). We denote them by Δ_k^s ($k = \overline{1,K}$). Then, the conditions under which the inequalities

$$|\delta\Phi_k| \leqslant \Delta_k^s \tag{10}$$

are satisfied can be conventionally considered as ecologically safe, and conditions under which these inequalities are violated are situations of environmental risk.

It follows from the sensitivity relations (8) that verification of the "ecological safety" inequalities (10) does not involve principal difficulties if the sensitivity functions are calculated and quantitative information on variations of parameters is available. Indeed, in the case of deterministic variations of sources and parameters, the range of variations of the functionals can be calculated by the formulas

$$|\delta\Phi_k| \leqslant \sum_{i=1}^{N} |\Gamma_{ki}|\,|\delta Y_i|. \tag{11}$$

The estimates of variations of the functionals in the case of random variations of parameters and sources are somewhat more complicated as compared to the deterministic variant of disturbances, since one has to operate with multidimensional spaces of SF and parameters. We consider one approach to obtaining the required estimates in the deterministic-stochastic case on the basis of the methods of the sensitivity theory [12] and mathematical theory of risks [13].

The calculations are organized so that the sensitivity vectors depend only on undisturbed values of parameters and the vector of state. Therefore, Γ_k can be assumed to be nonrandom for particular situations. Taking into account the properties of mathematical expectation and covariational matrices under linear transformations of random vectors, we obtain the following estimates for the mathematical expectation $E(\delta\Phi) = \sum\limits_{i=1}^{N} \Gamma_i E(\delta Y_i)$ and for the dispersion $D(\delta\Phi) = (D(\delta\boldsymbol{Y})\boldsymbol{\Gamma}, \boldsymbol{\Gamma})$ of variation of the functional $\delta\Phi$. In accordance with (8), variation of the functional $\delta\Phi$ is now determined as a linear combination of random quantities. If the number N is sufficiently large and there are no components of the vector $\delta\boldsymbol{Y}$ that strongly differ from the normal distribution, we can assume, on the basis of the central limit theorem of the probability theory [14], that the distribution law of the quantity $\delta\Phi$ tends to the normal one. The assumption on the normality of the distribution law significantly simplifies the problem, because to obtain a comprehensive characteristic of this distribution, one has to know its mathematical expectation $E(\delta\Phi)$ and dispersion $D(\delta\Phi)$ or the corresponding covariational matrix.

Using the values of $E(\delta\Phi)$ and $D(\delta\Phi)$ and the assumption on the normal distribution law for $\delta\Phi$ as a random quantity with the distribution density $f(x) = \mathrm{e}^{-(x-E(x))^2/2D(x)} / \sqrt{2\pi D(x)}$ $(x \equiv \delta\Phi)$, we can obtain a number of numerical characteristics of the estimate of the functional $\Phi(\boldsymbol{\varphi})$. The technique for constructing these estimates has been well developed [15]. We use the results of [6, 12].

In particular, it is of interest to consider the probability of satisfaction of inequalities (11) expressing the conditions under which the situation under study can be considered as ecologically safe

$$R^s = P(|\delta\Phi| \leqslant \Delta^s), \tag{12}$$

where P is the probability that the normally distributed quantity $\delta\Phi$ with a density $f(x)$ is located within the safety interval Δ^s.

If we define a certain acceptable level of the reliability probability R^s, we can determine the parameters of "reliability" λ of estimate (12) and calculate the range of "safety"

$$|\Delta^s - E(\delta\Phi)| = \lambda\sqrt{D(\delta\Phi)}.$$

In problems of ecological design [10], in addition to estimating the situation as a whole, one has to consider possible worst scenarios for the atmosphere quality in the receptor zone [11]. For this purpose, regions of local maximums of the calculated sensitivity functions and regions of sources with a high potential power (in the sense of emission of pollutants) are identified. If these regions coincide, situations of high environmental risk/vulnerability can emerge. In such cases, more detailed studies are required, e.g., scenarios of forward modeling with a given set of sources and different variants of emissions.

Basic Models of Atmosphere Hydrodynamics and Transport and Transformation of Pollutants. For practical implementation of the proposed concept of ecological prediction, the structure and principle of construction of the set of models of the atmosphere of an industrial region were developed; they are aimed at investigating the formation of mesoclimates in the region on the background of large-scale processes and also processes of pollutant transfer and transformation. In this concept, the region is considered as an element of the global climatic system, which acts both as a source of pollutants and their receptor. The West Siberian and East Siberian regions of the Siberian Federal District were chosen as objects of the study.

In accordance with the accepted concept, the set of models is constructed on the hierarchic principle so that basic models of several scales were involved: local, mesoscale, regional, hemispherical, and global models of the atmosphere. Each model includes a description of the processes of hydrothermodynamics and transport and transformation of pollutants from natural and man-caused sources. To organize interaction of the models with observed data and to set quality criteria and restrictions imposed on the solutions, we introduce functionals of generalized characteristics with the Radon and Dirac measures, which are determined on the sets of functions of state and measured data. A consistent description of all models and processes in terms of the functional content and construction of an adequate structure of numerical schemes and computational algorithms is ensured by the variational principle. Its application offers a single structural basis for the entire technological chain of mathematical modeling: from constructing discrete analogs of the models of considered processes to system organization of algorithms of forward and inverse modeling and optimization procedures. As a consequence of the variational principle in studying the sensitivity of models to variations of the input data and obtaining optimal estimates of functionals determined on the set of functions of state of these models, adjoint problems for models of the processes are obtained. The relations of the sensitivity theory are multipurpose and rather useful tools for studying models and processes. From the viewpoint of ecological safety, they provide quantitative information for evaluating the degree of environmental risks for the region-receptor of its own and "alien" pollutants; from the viewpoint of monitoring, these relations show the information value of the observation system and the possibility of identification of the sources of man-caused factors. In methods of inverse modeling, they are structural elements in the feedback from the objective functionals to the sought characteristics.

Formation of Long-Term Scenarios for Estimating Ecological Prospects. Problems of ecological prediction have another specific feature, namely: they have always to be solved with allowance for uncertainties in the behavior of the climatic system and in the character of human activities. If we accept the hypothesis of relative stability of the climate within 50–100 years, we can use the scenario approach for ecological predictions. The hypothesis of relative stability of the climate implies that the climatic changes, if any, occur gradually, and the range of variations is much smaller than the values of the main climatic parameters averaged over many years. In the scenario-approach strategy, one has to choose a suitable set of scenarios, which takes into account typical and extreme situations from the viewpoint of both climatic changes and possible directions of economic activity. For each scenario, its own ecological prediction is made, and the set of scenarios is analyzed as a whole by methods of the factor analysis. Note, implementation of a series of scenario predictions allows the most effective use of advanced technologies of parallel computations.

Methods of the factor analysis extend the possibilities of mathematical models in studying natural processes [16, 17]. They allow one to present the set of data on long-term dynamics of the considered processes as a full system of orthogonal subspaces in decreasing order of their significance level in accordance with specified criteria [18]. The most informative components of these subspaces are usually classified as principal factors.

The following problems are effectively solved with the help of the principal factors:

— representation of the initial set of vectors with a specified significance level by a small number of components; investigation of annual and seasonal variability of the processes;

— identification of active zones in the climatic system, identification of their spatial position and investigation of variability; investigation of their relations to regions of environmental risks/vulnerability;

— typification of long-term dynamics of the examined system in accordance with the intensity of factor loads with respect to the principal components; identification of typical and anomalous situations;

— formation of informative phase spaces for organizing deterministic-stochastic scenarios on the basis of models of hydrodynamics and transport of pollutants.

The use of information on the principal factors in ecological studies allows one to effectively choose methods for the description of meteorological situations, which form the background for long-term processes of transport and transformation of moisture and pollutants. An analysis of the principal factors of the global and regional scales together with an analysis of the sensitivity functions and influence regions for models and functionals shows that it is impossible to adequately reproduce the dynamics of regional processes without taking into account their interrelations with global processes. This is clearly manifested, e.g., in estimating the environmental risks and vulnerability to man-caused factors for the region under study.

Based on these postulates, the modeling technology is organized on the principles of complexation of results obtained by models of different scales and decomposition of the function of state over scales into background processes and disturbances. The essence of this technology is as follows. New elements (guiding phase spaces) are introduced into the model. These are multicomponent fields of the space–time structure, which offer a description of background processes with a specified information value relative to the observed states of the global climatic system in accordance with typification by the principal factors. To identify subspaces of the principal factors and form the guiding phase spaces, we use the NCEP/NCAP reanalysis database, which contains multidimensional dynamics of the global climatic system from 1950 to 2002 with a discreteness of 12 hours [19].

The detailed field structure is calculated by a set of mathematical models. The global and regional models are simultaneously used for regional studies [8]. Complexation of models of different scales and allowance for the guiding phase spaces are performed in real time at each time step with the help of the variational principle of minimization of the total measure of uncertainties in models and data. It is important here that all information arrays are presented in grid domains correlated in structure. This simplifies the nesting procedure without losing the information content and accuracy of field presentation.

Estimation of the Risk for Lake Baikal. The set of models developed is used to solve research and applied problems on estimation of the ecological prospects of industrial regions.

We give an example of an inverse modeling scenario. The objective is to evaluate the degree to which Lake Baikal and surrounding territories in the region are subjected to the action of man-caused factors and to estimate the environmental risk for the lake to receive pollutants from the acting and potentially possible sources. Using the solutions of adjoint problems in the inverse mode, we calculate the sensitivity functions of the atmosphere quality functional. This functional is formed as follows. The region above the lake is defined as a support of nonzero values of the weight function in (7) and $F_k(\boldsymbol{\varphi}) \equiv \boldsymbol{\varphi}$. The function $\boldsymbol{\varphi}$ describes the concentration of passive pollutants in the

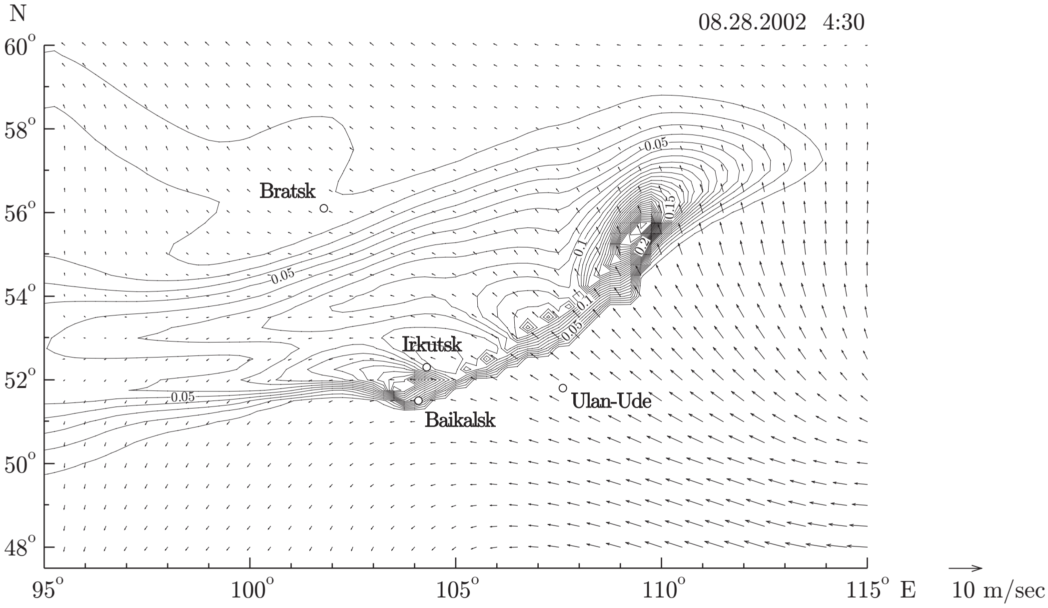

Fig. 1.

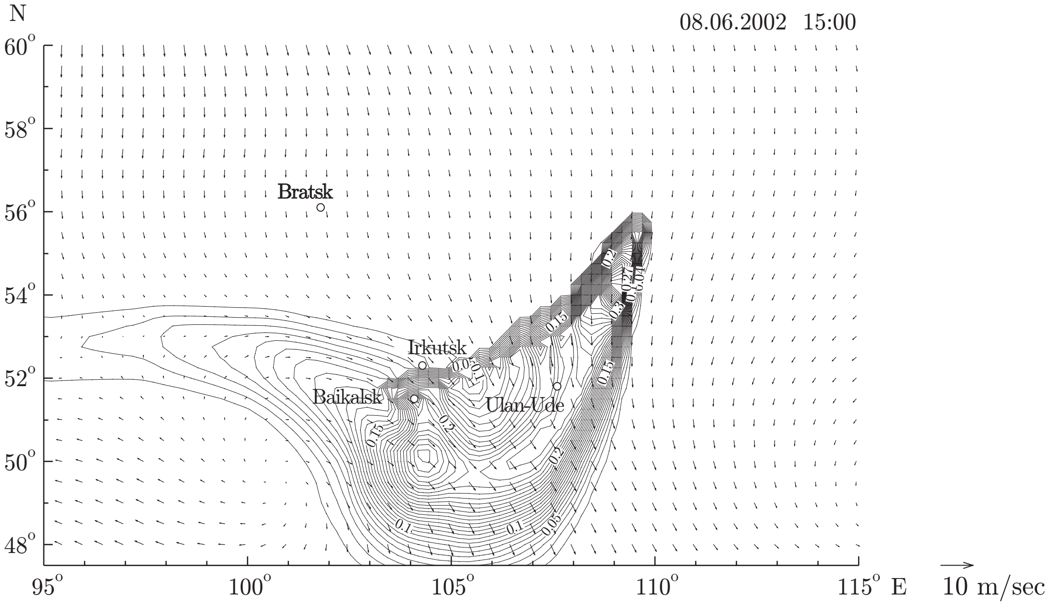

Fig. 2.

region. The functional shows the total amount of pollutants, which can enter the atmosphere above the lake during a month from the acting and potentially possible sources. The scenario is calculated for August 2002. Scenario hydrodynamics is formed with the use of the data of [19]. The dynamics of four-dimensional fields of meteoelements is calculated from August 1 to August 31, 2002 with a time step of 30 min in the region of 47.5–60° N and 95–115° E on a latitude–longitude grid with $15'$ steps in each direction. The vertical resolution is 19 levels in hybrid coordinates [8] from the Earth surface to the level corresponding to a pressure of 10 mbar.

Figures 1 and 2 show the velocity fields at the upper boundary of the surface layer (arrows) and sensitivity functions to variations of the power of sources located on the Earth surface in the region and outside it (relative units). The SF is a four-dimensional aggregate of the space–time structure. The figures show the two-dimensional fragments that refer to two time moments: August 28 (Fig. 1) and August 6 (Fig. 2) The SF values show which part of the total emission from the acting and potentially possible sources can enter the near-water layer of the atmosphere. The higher the SF value, the greater the risk of receiving pollution. The figures demonstrate high variability of the sensitivity functions in space and time. Thus, it is seen in Fig. 1 that pollutants can appear from the north-west direction. It is of interest that there are centers of local maximums in regions characterized by a high level of man-caused factors: Irkutsk, Angarsk, etc. In Fig. 2, the risk zones are located both at the north-west and at the south-east, which can be related to the influence of the Baikal pulp mill and other industrial objects of this region. In addition, significant part of the risk zone belongs to Mongolia. Analyzing the Baikal scenarios, we can conclude that the risk zone for the lake is determined by the industrially loaded territories of the West-Siberian region.

The information gained from sensitivity functions can be beneficial for planning economic activity and for ecological prediction and design. Note one more useful application of sensitivity functions for organizing systems of monitoring of natural environment quality. The geometric configuration of the SF support is a characteristic of observability of the region territories with the help of measurement systems located in receptor regions. In this case, the SF values are the measure of the information value of observations relative to the sources. This means that, based on observations from the receptor region, solutions of inverse problems, and calculated sensitivity functions, one can find the locations of the sources of specific pollutants and identify their parameters.

Conclusions. A technique is proposed for calculating environmental risks and vulnerability on the basis of variational principles and combination of methods of forward and inverse modeling, methods of the sensitivity theory, and multidimensional factor analysis. The technique can be readily adapted to the analysis of particular situations. To obtain a solution of the risk/vulnerability problem from the viewpoint of ecological prediction, however, one has to take into account the features of the real behavior of the climatic system and uncertainties inherent in this class of problems. For these purposes, a scenario approach with the use of a set of models and methods of multidimensional and multicomponent factor analysis is suggested, which allows one to identify the most important, typical, and extreme situations in the regime of annual and seasonal variability.

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